

Analyzing Graphs of Motion With Numbers



Speed can be calculated from position-time graphs and distance can be calculated from speed-time graphs. Both calculations rely on the familiar speed equation: $v = d/t$.

This graph shows position and time for a sailboat starting from its home port as it sailed to a distant island. By studying the line, you can see that the sailboat traveled 10 miles in 2 hours.

EXAMPLES

• Calculating speed from a position-time graph

The speed equation allows us to calculate that the boat's speed during this time was 5 miles per hour.

$$v = d/t$$

$$v = 10 \text{ miles}/2 \text{ hours}$$

$$v = 5 \text{ miles/hour, read as 5 miles per hour}$$

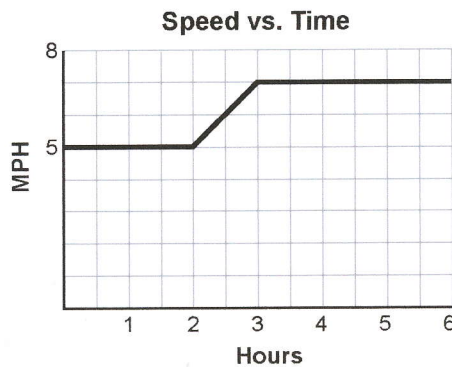
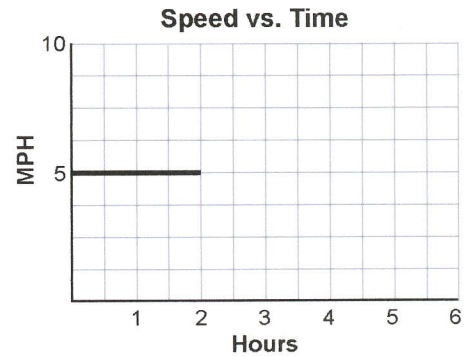
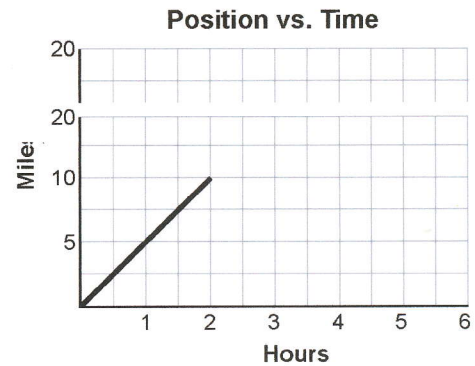
This result can now be transferred to a speed-time graph. Remember that this speed was measured during the first two hours.

The line showing the boat's speed is horizontal because the speed was constant during the two-hour period.

• Calculating distance from a speed-time graph

Here is the speed-time graph of the same sailboat later in the voyage. Between the second and third hours, the wind freshened and the sailboat gradually increased its speed to 7 miles per hour. The speed remained 7 miles per hour to the end of the voyage.

How far did the sailboat go during the six-hour trip? We will first calculate the distance traveled between the third and sixth hours.



On a speed-time graph, distance is equal to the area between the baseline and the plotted line. You know that the area of a rectangle is found with the equation: $A = L \times W$. Similarly, multiplying the speed from the y -axis by the time on the x -axis produces distance. Notice how the labels cancel to produce miles:

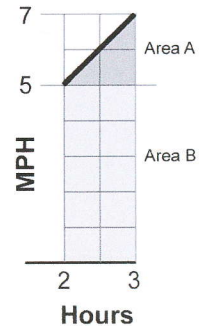
$$\text{speed} \times \text{time} = \text{distance}$$

$$7 \text{ miles/hour} \times (6 \text{ hours} - 3 \text{ hours}) = \text{distance}$$

$$7 \text{ miles/hour} \times 3 \text{ hours} = \text{distance} = 21 \text{ miles}$$

Now that we have seen how distance is calculated, we can consider the distance covered between hours 2 and 3.

The easiest way to visualize this problem is to think in geometric terms. Find the area of the triangle (Area A), then find the area of the rectangle (Area B), and add the two areas.



Area of triangle A
Geometry formula

The area of a triangle is one-half the area of a rectangle.

$$\text{speed} \times \frac{\text{time}}{2} = \text{distance}$$

$$(7 \text{ miles/hour} - 5 \text{ miles/hour}) \times \frac{(3 \text{ hours} - 2 \text{ hours})}{2} = \text{distance} = 1 \text{ mile}$$

Area of rectangle B
Geometry formula

$$\text{speed} \times \text{time} = \text{distance}$$

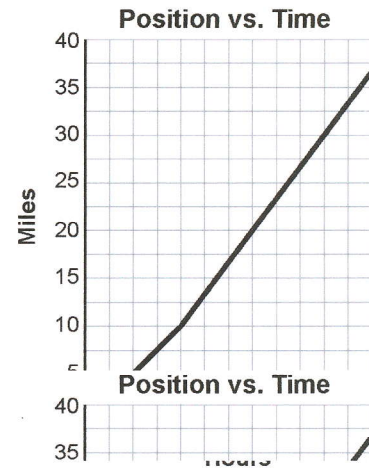
$$5 \text{ miles/hour} \times (3 \text{ hours} - 2 \text{ hours}) = \text{distance} = 5 \text{ miles}$$

Add the two areas

$$\text{Area A} + \text{Area B} = \text{distance}$$

$$1 \text{ miles} + 5 \text{ mile} = \text{distance} = 6 \text{ miles}$$

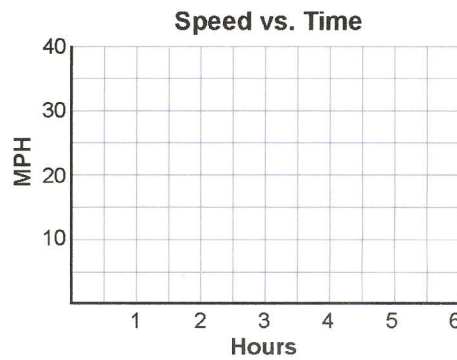
We can now take the distances found for both sections of the speed graph to complete our position-time graph:



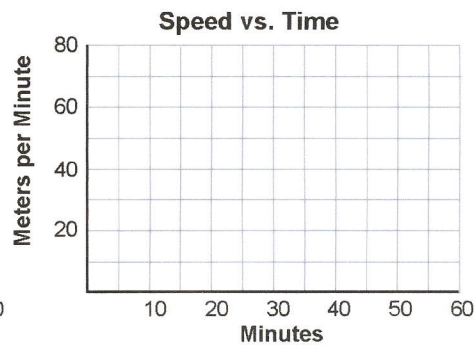
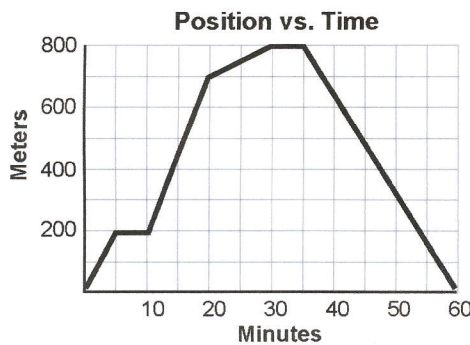
PRACTICE 

1. For each position-time graph, calculate and plot speed on the speed-time graph to the right.

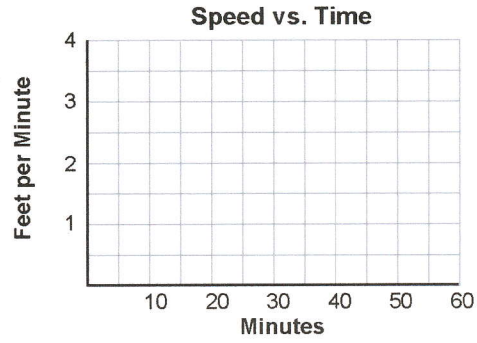
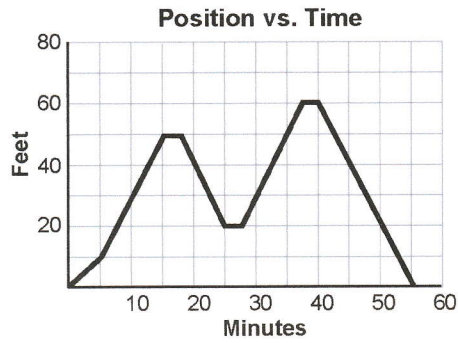
a. The bicycle trip through hilly country



b. A walk in the park

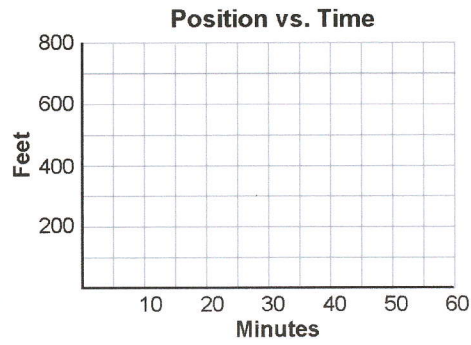
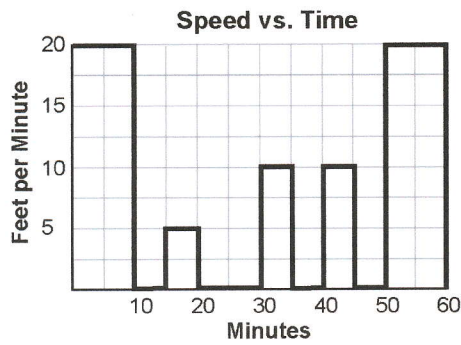


c. Strolling up and down the supermarket aisles

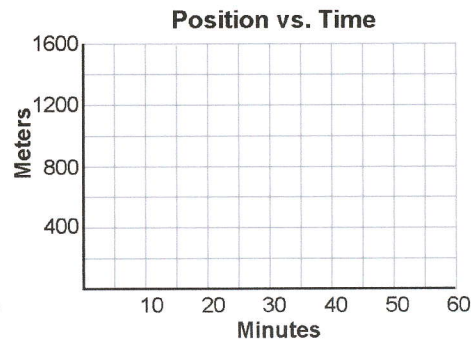
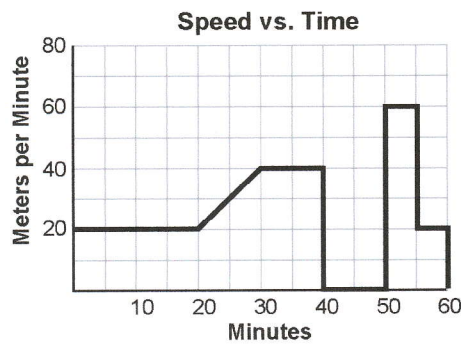


2. For each speed-time graph, calculate and plot the distance on the position-time graph to the right. For this practice, assume that movement is always away from the starting position.

a. The honey bee among the flowers



b. Rover runs the street



c. The amoeba

